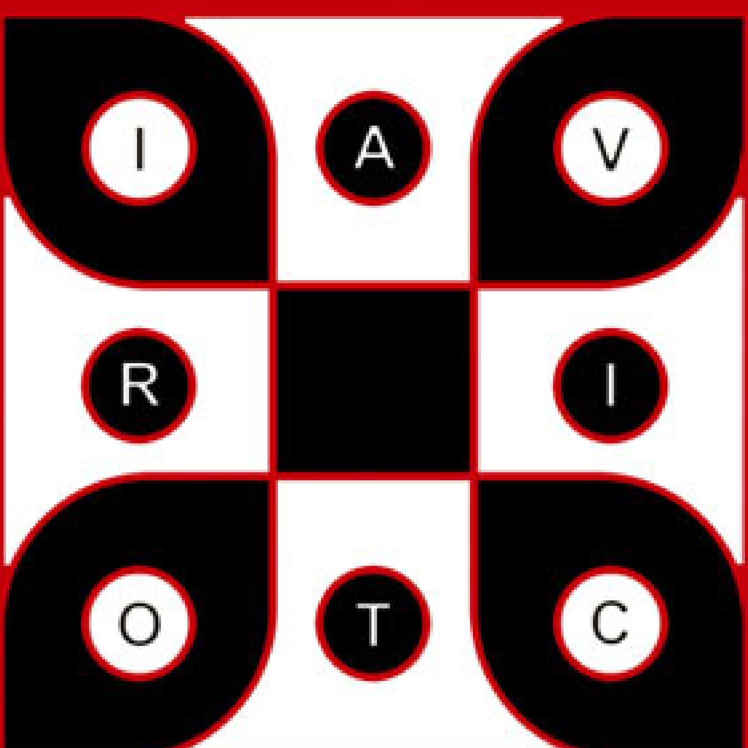
Good old-fashioned

Challenging Puzzles

and perplexing mathematical problems



H. E. Dudeney

GOOD OLD-FASHIONED CHALLENGING PUZZLES
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Summersdale Publishers Ltd
46 West Street
Chichester
West Sussex
PO19 1RP
UK

www.summersdale.com

Printed and bound in Great Britain.

ISBN: 1-84024-557-3
ISBN 13: 978-1-84024-557-8

EDITOR'S NOTE

Henry Ernest Dudeney (1857–1930) was an author and mathematician, and is known as one of the country's foremost creators of puzzles. His puzzles regularly appeared in newspapers and magazines.

Good Old-fashioned Challenging Puzzles is a selection of mathematical brain-teasers from his book *Amusements in Mathematics*, first published in 1917 and hailed by *The Spectator* as 'not only an amusement but a revelation'. Some of the problems are, as Dudeney admitted, 'not unworthy of the attention of the advanced mathematician'.

For today's lovers of sudoku, chess, poker, bridge, cryptic crosswords and other brain-training games, these good old-fashioned puzzles and problems will provide many hours of amusement and an unparalleled opportunity to exercise your logical and mathematical agility. We have chosen to leave Dudeney's uniquely witty preface and introductions largely as they were when the book first appeared.

PREFACE

In issuing this volume of my Mathematical Puzzles, of which some have appeared in periodicals and others are given here for the first time, I must acknowledge the encouragement that I have received from many unknown correspondents, at home and abroad, who have expressed a desire to have the problems in a collected form, with some of the solutions given at greater length than is possible in magazines and newspapers. Though I have included a few old puzzles that have interested the world for generations, where I felt that there was something new to be said about them, the problems are in the main original.

On the question of Mathematical Puzzles in general there is, perhaps, little more to be said than I have written elsewhere. The history of the subject entails nothing short of the actual story of the beginnings and development of exact thinking in man. The historian must start from the time when man first succeeded in counting his ten fingers and in dividing an apple into two approximately equal parts. Every puzzle that is worthy of consideration can be referred to mathematics and logic. Every man, woman, and child who tries to 'reason out' the answer to the simplest puzzle is working, though not of necessity consciously, on mathematical lines. Even those puzzles that we have no way of attacking except by haphazard attempts can be brought under a method of what has been called 'glorified trial'—a system of shortening our labours by avoiding or eliminating what our reason tells us is useless. It is, in fact, not easy to say sometimes where the 'empirical' begins and where it ends.

When a man says, 'I have never solved a puzzle in my life,' it is difficult to know exactly what he means, for every intelligent individual is doing it every day. The unfortunate inmates of our lunatic asylums are sent there expressly because they cannot solve puzzles—because they have lost their powers of reason. If there were no puzzles to solve, there would be no questions to ask; and if there were no questions to be asked, what a world it would be! We should all be equally omniscient, and conversation would be useless and idle.

It is possible that some few exceedingly sober-minded mathematicians, who are impatient of any terminology in their favourite science but the academic, and who object to the elusive x and y appearing under any other names, will have wished that various problems had been presented in a less popular dress and introduced with a less flippant phraseology. I can only refer them to the first word of my title and remind them that we are primarily out to be amused—not, it is true, without some hope of picking up morsels of knowledge by the way. If the manner is light, I can only say, in the words of Touchstone, that it is 'an ill-favoured thing, sir, but my own; a poor humour of mine, sir.'

As for the question of difficulty, some of the puzzles, especially in the Arithmetical and Algebraical category, are quite easy. Yet some of those examples that look the simplest should not be passed over without a little consideration, for now and again it will be found that there is some more or less subtle pitfall or trap into which the reader may be apt to fall. It is good exercise to cultivate the habit of being very wary over the exact wording of a puzzle. It teaches exactitude and caution. But some of the problems are very hard nuts indeed, and not unworthy of the attention of the advanced mathematician. Readers will doubtless select according to their individual tastes.

In many cases only the mere answers are given. This leaves the beginner something to do on his own behalf in working out the method of solution, and saves space that would be wasted from the point of view of the advanced student. On the other hand, in particular cases where it seemed likely to interest, I have given rather extensive solutions and treated problems in a general manner. It will often be found that the notes on one problem will serve to elucidate a good many others in the book; so that the reader's difficulties will sometimes be found cleared up as he advances. Where it is possible to say a thing in a manner that may be 'understanded of the people' generally, I prefer to use this simple phraseology, and so engage the attention and interest of a larger public. The mathematician will in such cases have no difficulty in expressing the matter under consideration in terms of his familiar symbols.

I have taken the greatest care in reading the proofs, and trust that any errors that may have crept in are very few. If any such should occur, I can only plead, in the words of Horace, that 'good Homer sometimes nods,' or, as the bishop put it, 'Not even the youngest curate in my diocese is infallible.'

I have to express my thanks in particular to the proprietors of *The Strand Magazine*, *Cassell's Magazine*, *The Queen*, *Tit-Bits*, and *The Weekly Dispatch* for their courtesy in allowing me to reprint some of the puzzles that have appeared in their pages.

THE AUTHORS' CLUB
March 25, 1917

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ARITHMETICAL AND ALGEBRAICAL PROBLEMS

'And what was he?
Forsooth, a great arithmetician.'

Othello, I. i.

The puzzles in this department are roughly thrown together in classes for the convenience of the reader. Some are very easy, others quite difficult. But they are not arranged in any order of difficulty—and this is intentional, for it is well that the solver should not be warned that a puzzle is just what it seems to be. It may, therefore, prove to be quite as simple as it looks, or it may contain some pitfall into which, through want of care or over-confidence, we may stumble.

Also, the arithmetical and algebraical puzzles are not separated in the manner adopted by some authors, who arbitrarily require certain problems to be solved by one method or the other. The reader is left to make his own choice and determine which puzzles are capable of being solved by him on purely arithmetical lines.

MONEY PUZZLES

'Put not your trust in money, but put your money in trust.'

OLIVER WENDELL HOLMES.

1. AT A CATTLE MARKET.

Three countrymen met at a cattle market. 'Look here,' said Hodge to Jakes, 'I'll give you six of my pigs for one of your horses, and then you'll have twice as many animals here as I've got.' 'If that's your way of doing business,' said Durrant to Hodge, 'I'll give you fourteen of my sheep for a horse, and then you'll have three times as many animals as I.' 'Well, I'll go better than that,' said Jakes to Durrant; 'I'll give you four cows for a horse, and then you'll have six times as many animals as I've got here.'

No doubt this was a very primitive way of bartering animals, but it is an interesting little puzzle to discover just how many animals Jakes, Hodge, and Durrant must have taken to the cattle market.

2. THE TWO AEROPLANES.

A man recently bought two aeroplanes, but afterwards found that they would not answer the purpose for which he wanted them. So he sold them for £600 each, making a loss of 20 per cent on one machine and a profit of 20 per cent on the other. Did he make a profit on the whole transaction, or a loss? And how much?

3. THE MILLIONAIRE'S PERPLEXITY.

Mr. Morgan G. Bloomgarten, the millionaire, known in the States as the Clam King, had, for his sins, more money than he knew what to do with. It bored him. So he determined to persecute some of his poor but happy friends with it. They had never done him any harm, but he resolved to inoculate them with the 'source of all evil.' He therefore proposed to distribute a million dollars among them and watch them go rapidly to the bad. But he was a man of strange fancies and superstitions, and it was an inviolable rule with him never to make a gift that was not either one dollar or some power of seven—such as 7, 49, 343, 2,401, which numbers of dollars are produced by simply multiplying sevens together. Another rule of his was that he would never give more than six persons exactly the same sum. Now, how was he to distribute the 1,000,000 dollars? You may distribute the money among as many people as you like, under the conditions given.

4. THE GROCER AND DRAPER.

A country 'grocer and draper' had two rival assistants, who prided themselves on their rapidity in serving customers. The young man on the grocery side could weigh up two one-pound parcels of sugar per minute, while the drapery assistant could cut three one-yard lengths of cloth in the same time. Their employer, one slack day, set them a race, giving the grocer a barrel of sugar and telling him to weigh up forty-eight one-pound parcels of sugar while the draper divided a roll of forty-eight yards of cloth into yard pieces. The two men were interrupted together by customers for nine minutes, but the draper was disturbed seventeen times as long as the grocer. What was the result of the race?

5. JUDKINS'S CATTLE.

Hiram B. Judkins, a cattle-dealer of Texas, had five droves of animals, consisting of oxen, pigs, and sheep, with the same number of animals in each drove. One morning he sold all that he had to eight dealers. Each dealer bought the same number of animals, paying seventeen dollars for each ox, four dollars for each pig, and two dollars for each sheep; and Hiram received in all three hundred and one dollars. What is the greatest number of animals he could have had? And how many would there be of each kind?

AGE AND KINSHIP PUZZLES

'The days of our years are threescore years and ten.'
—*Psalm* xc. 10.

For centuries it has been a favourite method of propounding arithmetical puzzles to pose them in the form of questions as to the age of an individual. They generally lend themselves to very easy solution by the use of algebra, though often the difficulty lies in stating them correctly. They may be made very complex and may demand considerable ingenuity, but no general laws can well be laid down for their solution. The solver must use his own sagacity. As for puzzles in relationship or kinship, it is quite curious how bewildering many people find these things. Even in ordinary conversation, some statement as to relationship, which is quite clear in the mind of the speaker, will immediately tie the brains of other people into knots. In such cases the best course is to sketch a brief genealogical table, when the eye comes immediately to the assistance of the brain. In these days, when we have a growing lack of respect for pedigrees, most people have got out of the habit of rapidly drawing such tables, which is to be regretted, as they would save a lot of time and brain racking on occasions.

6. MAMMA'S AGE.

Tommy: 'How old are you, mamma?'
Mamma: 'Let me think, Tommy. Well, our three ages add up to exactly seventy years.'
Tommy: 'That's a lot, isn't it? And how old are you, papa?'
Papa: 'Just six times as old as you, my son.'
Tommy: 'Shall I ever be half as old as you, papa?'
Papa: 'Yes, Tommy; and when that happens our three ages will add up to exactly twice as much as to-day.'
Tommy: 'And supposing I was born before you, papa; and supposing mamma had forgot all about it, and hadn't been at home when I came; and supposing—'
Mamma: 'Supposing, Tommy, we talk about bed. Come along, darling. You'll have a headache.'

Now, if Tommy had been some years older he might have calculated the exact ages of his parents from the information they had given him. Can you find out the exact age of mamma?

7. THEIR AGES.

'My husband's age,' remarked a lady the other day, 'is represented by the figures of my own age reversed. He is my senior, and the difference between our ages is one-eleventh of their sum.'

8. ROVER'S AGE.

'Now, then, Tommy, how old is Rover?' Mildred's young man asked her brother.

'Well, five years ago,' was the youngster's reply, 'sister was four times older than the dog, but now she is only three times as old.'

Can you tell Rover's age?

9. CONCERNING TOMMY'S AGE.

Tommy Smart was recently sent to a new school. On the first day of his arrival the teacher asked him his age, and this was his curious reply: 'Well, you see, it is like this. At the time I was born—I forget the year—my only sister, Ann, happened to be just one-quarter the age of mother, and she is now one-third the age of father.' 'That's all very well,' said the teacher, 'but what I want is not the age of your sister Ann, but your own age.' 'I was just coming to that,' Tommy answered; 'I am just a quarter of mother's present age, and in four years' time I shall be a quarter the age of father. Isn't that funny?'

This was all the information that the teacher could get out of Tommy Smart. Could you have told, from these facts, what was his precise age? It is certainly a little puzzling.

10. THE BAG OF NUTS.

Three boys were given a bag of nuts as a Christmas present, and it was agreed that they should be divided in proportion to their ages, which together amounted to 17½ years. Now the bag contained 770 nuts, and as often as Herbert took four Robert took three, and as often as Herbert took six Christopher took seven. The puzzle is to find out how many nuts each had, and what were the boys' respective ages.

11. A FAMILY PARTY.

A certain family party consisted of 1 grandfather, 1 grandmother, 2 fathers, 2 mothers, 4 children, 3 grandchildren, 1 brother, 2 sisters, 2 sons, 2 daughters, 1 father-in-law, 1 mother-in-law, and 1 daughter-in-law. Twenty-three people, you will say. No; there were only seven persons present. Can you show how this might be?

CLOCK PUZZLES

'Look at the clock!'

Ingoldsby Legends.

In considering a few puzzles concerning clocks and watches, and the times recorded by their hands under given conditions, it is well that a particular convention should always be kept in mind. It is frequently the case that a solution requires the assumption that the hands can actually record a time involving a minute fraction of a second. Such a time, of course, cannot be really indicated. Is the puzzle, therefore, impossible of solution? The conclusion deduced from a logical syllogism depends for its truth on the two premises assumed, and it is the same in mathematics. Certain things are antecedently assumed, and the answer depends entirely on the truth of those assumptions.

'If two horses,' says Lagrange, 'can pull a load of a certain weight, it is natural to suppose that four horses could pull a load of double that weight, six horses a load of three times that weight. Yet, strictly speaking, such is not the case. For the inference is based on the assumption that the four horses pull alike in amount and direction, which in practice can scarcely ever be the case. It so happens that we are frequently led in our reckonings to results which diverge widely from reality. But the fault is not the fault of mathematics; for mathematics always gives back to us exactly what we have put into it. The ratio was constant according to that supposition. The result is founded upon that supposition. If the supposition is false the result is necessarily false.'

If one man can reap a field in six days, we say two men will reap it in three days, and three men will do the work in two days. We here assume, as in the case of Lagrange's horses, that all the men are exactly equally capable of work. But we assume even more than this. For when three men get together they may waste time in gossip or play; or, on the other hand, a spirit of rivalry may spur them on to greater diligence. We may assume any conditions we like in a problem, provided they be clearly expressed and understood, and the answer will be in accordance with those conditions.

12. A TIME PUZZLE.

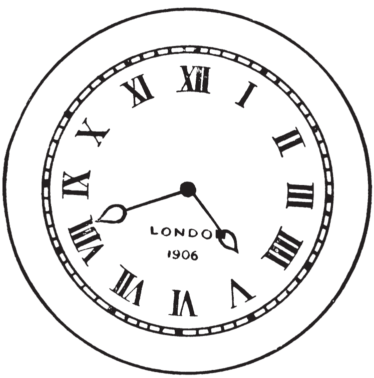
How many minutes is it until six o'clock if fifty minutes ago it was four times as many minutes past three o'clock?

13. THE WAPSHAW'S WHARF MYSTERY.

There was a great commotion in Lower Thames Street on the morning of January 12, 1887. When the early members of the staff arrived at Wapshaw's Wharf they found that the safe had been broken open, a considerable sum of money removed, and the offices left in great disorder. The night watchman was nowhere to be found, but nobody who had been acquainted with him for one moment suspected him to be guilty of the robbery. In this belief the proprietors were confirmed when, later in the day, they were informed that the poor fellow's body had been picked up by the River Police. Certain marks of violence pointed to the fact that he had been brutally attacked and thrown into the river. A watch found in his pocket had stopped, and this was a valuable clue to the time of the outrage. But a very stupid officer had actually amused himself by turning the hands round and round, trying to set the watch going again. After he had been severely reprimanded for this serious indiscretion, he was asked whether he could remember the time that was indicated by the watch when found. He replied that he could not, but he recollected that the hour hand and minute hand were exactly together, one above the other, and the second hand had just passed the forty-ninth second. More than this he could not remember.

What was the exact time at which the watchman's watch stopped? The watch is, of course, assumed to have been an accurate one.

14. CHANGING PLACES.



The clock face indicates a little before 42 minutes past 4. The hands will again point at exactly the same spots a little after 23 minutes past 8. In fact, the hands will have changed places. How many times do the hands of a clock change places between three o'clock p.m. and midnight? And out of all the pairs of times indicated by these changes, what is the exact time when the minute hand will be nearest to the point IX?

15. THE STOP-WATCH.

We have here a stop-watch with three hands. The second hand, which travels once round the face in a minute, is the one with the little ring at its end near the centre. Our dial indicates the